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EFFECT OF FIXED TIME DELAY ON STABILITY AND PERFORMANCE OF ACTIVELY CONTROLLED CIVIL ENGINEERING STRUCTURES

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SUMMARY

In this paper, a state-of-the-art review for the fixed time delay of actively controlled civil engineering structures is presented, including the identification of time delay, the effect of time delay on the stability and performance of the controlled structures, and the evaluation of the critical time delay. In particular, a critical review of the stability analysis methods currently available for the critical time delay of Multiple-Degree-of-Freedom (MDOF) systems is conducted and new simulation results are presented to show its limitation in practical applications. Further, a method of stability analysis for the critical time delay of MDOF systems equipped with single or multiple actuators is presented along with the simulation results to demonstrate its applications to seismic hazard mitigations. Under earthquake excitations, simulation results for the structural response indicate that the degradation of the control performance due to the fixed time delay is not significant until the time delay is close to the critical time delay. It is further demonstrated that the time-delay problem is more serious for structures with closely spaced vibrational modes, such as a building equipped with an active tuned mass damper. © 1997 John Wiley & Sons, Ltd.

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KEY WORDS: structural control; system time delay; system stability; structural dynamics; seismic response

INTRODUCTION

Active control of civil engineering structures has been investigated theoretically and experimentally and different active control systems have been installed in tall buildings in Japan (e.g. Reference 1). One important issue of structural control is the time-delay problem. Figure 1 shows the schematic diagram of different components of a structural control system, e.g. sensors, filters, computers, actuators, etc. The entire control process involves measuring vibrational data, conditioning and filtering of these data, computing the control forces, transmitting data and signals to actuators, applying control forces to the structure, etc. (e.g. Reference 2). This process results in a time delay in applying the required control forces to the structure. Applications of unsynchronized control forces due to time delay may result in a degradation of the control performance and may even render the controlled structure to be unstable. Hence, the problem of time delay has been studied quite extensively in the literature.

The objectives of this paper are to (i) conduct a literature survey on the effect of time delay for applications to civil engineering structures subject to natural hazards, such as earthquakes and wind gusts, (ii) perform a critical evaluation of available analysis methods for determining the critical time delay for Multiple-Degrees-of-Freedom (MODF) systems and (iii) present a new analysis methodology for determining the

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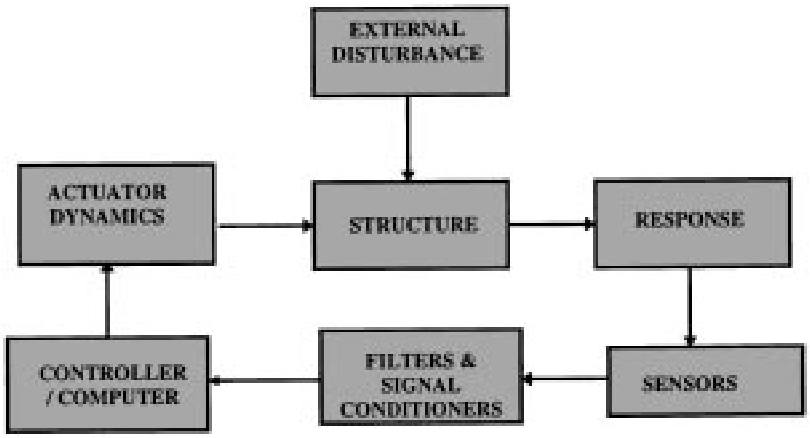


Figure 1. Schematic diagram showing various components of active control process

critical time delay for MDOF structures. The literature review includes the identification of time delay, the effect of time delay on the stability and performance of controlled structures, and the evaluation of the critical time delay. The time delay that will render the controlled structure to be unstable is referred to as the critical time delay. Most of the analysis methods available in the literature for the determination of the critical time delay are restricted to Single-Degree-of-Freedom (SDOF) structures. Unfortunately, civil engineering structures are not only MDOF but also complex, involving many degrees of freedom. As such, a critical evaluation of the approximate method available in the literature for MDOF structures is conducted. The approximate method assumes the existence of normal modes (i.e. proportionally damped structures) and ignores the coupling effect among different modes due to the feedback control forces. Simulation results for this approach are presented to illustrate its applicability and limitations. Finally, a general method of analysis for the determination of the critical time-delay for MDOF structures is presented, along with simulation results to demonstrate its applications.

IDENTIFICATION OF FIXED TIME DELAY

In general, the total time delay in a control system is the sum of time delays due to (i) on-line data acquisition from sensors at different points of the structure, filtering, processing of data, calculating control forces and transmitting the control force signals from computer to the actuator (β_1), and (ii) the time taken by the actuator to build up required control force (β_2). Figure 2 shows the simplified representation of these time delays. The time delay β_1 is referred to as the fixed (or pure or dead) time delay, whereas the time delay β_2 depends on the particular dynamics of actuators. For example, Agrawal *et al.*[3] have shown that the time required for 98 and 99·99 per cent build-ups of the control force for an actuator that can be modelled by the first-order dynamics are $3{\cdot}912/\omega_a$ and $9{\cdot}21/\omega_a$, respectively, where ω_a is the actuator frequency. Besides structural control systems, fixed or true time delay occurs in other types of systems also, e.g. combustion, continuous systems described by partial differential equation (e.g. hyperbolic systems), process control, linear-control systems incorporating a digital computer, etc. In particular, the use of digital computers in control systems leads to true time delays, since it takes a finite time to carry out a computation. In general, such time delays vary with the data being processed. However, the effective computational time delay can be made constant and equal to the sampling period by using periodically driven sample and hold systems (e.g. Houspis and Lamont,[49] 1992). Usually, the sample and hold circuits are built into the Analog to Digital (A/D) and Digital to Analog (D/A) converters.

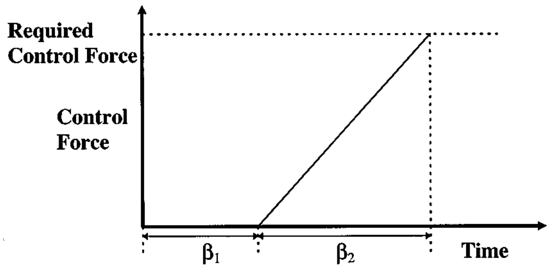


Figure 2. Time delays during an active control process

The representation of time delays in Figure 2 does not include the variation of time delay with the frequency. In practical applications, the fixed time delay may vary with the frequency content of the external excitation because of the frequency-dependent phase-lag characteristics of various components of the control system. Time delay in any component of a control system can be identified by measuring the phase lag between the input and the output signals at various frequencies as

$$\beta = \theta^\circ/(360f) \tag{1}$$

where θ° is the phase lag in degree and f is the frequency in Hertz. Reinhorn *et al.*[4] used this concept to identify the time delay experimentally. Similarly, Abdel-Mooty and Roorda[5] measured the fixed time delay experimentally for control of a simply supported beam equipped with a hydraulic actuator. The time delay in their experiments was attributed mainly to the actuator movement, cable stretching, filtering/conditioning, and analog integration. Carotti and Lio[6] presented the results of systematic tests to assess delays in all the stages of the control system. It was found that $\beta_1 \approx 50$ msec and $\beta_2 \approx 100$ msec for electrohydraulic actuators. However, since the time delay β_2 depends on the dominant frequency of the actuator, the results obtained by Carotti and Lio[6] are valid only for the particular experimental set-up used. Merritt[7] showed that the dynamics of electrohydraulic servo actuator is due to the interaction between trapped oil springs in the chambers of the cylinder and inertia loads the actuator is subject to. The fundamental frequency ω_a of the actuator, whose dynamics is characterized by a second-order equation, is given by

$$\omega_a = \sqrt{4\beta_e A^2/V_t M_t} \tag{2}$$

where β_e is the effective bulk modulus of the oil, A is the area of piston, V_t is the total volume of the oil under compression, and M_t is the total mass of the piston and the inertia loads on the piston. The actuator frequency ω_a may be of the order of 100–200 Hz for the actuator subject only to the inertia load of the piston mass. However, since M_t may be very large (approximately 1 per cent of the building weight) for control systems using active tuned mass damper (ATMD) or active mass driver (AMD), ω_a may fall near the lower frequencies of the building because of the inverse relationship between ω_a and M_t in equation (2). Consequently, β_2 will vary for different applications and it may be much longer than 100 msec in practical implementations of active control systems using ATMD, AMD or other controllers for civil engineering applications. Kobori *et al.*[8] investigated the installation of an AMD on an 11-storey building located in Kyobashi area of Tokyo. Their investigation showed that the experimental transfer function for the actuator dynamics is a second-order equation with a dominant frequency of 7·071 Hz and a damping ratio of 46 per cent. The mass of the damper in this case is 4·2 tons (1·05 per cent of the building weight). Using a unit step loading, it can be shown that the control force build-up time (the time for the response to reach unity before the peak overshoot) is 53 msec and the time to attain steady-state condition is approximately 200 msec. Similarly, Koike *et al.*[9] derived the transfer function of a DC motor servo control system experimentally for a proposed application of a hybrid mass damper on a 230 m tall building in Tokyo. The transfer function is

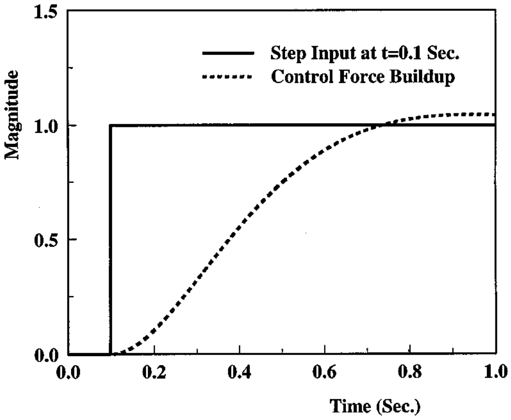


Figure 3. Control force buildup for an actuator with 0.82 Hz frequency and 70% damping ratio

the second-order equation with a frequency of 0.82 Hz and a damping ratio of 70 per cent. The mass of the damper is 330 tons (0.25 per cent of the building mass) and the actuator response subject to a unit-step loading is shown in Figure 3. It is observed from Figure 3 that the total build-up time delay, β_2, may be of the order of 650 msec.

The identification of time delay by Reinhorn *et al.*[4] and Carotti and Lio[6] was based on the direct experimental measurements of time delay due to each individual component of the control process. Based on an algorithm proposed by Fong-Chwee and Sirisena[10] for self-tunning PID control of deadtime processes, Abdel-Rohman *et al.*[11] presented an approach to identify the time delay for a discrete-time system, which minimizes an error vector of parameters in the system. The identified time delay is a multiple of sampling time, and the method does not consider the effect of the measurement noise. Abdel-Rohman *et al.*[11] showed the application of this method using an example of a SDOF system equipped with an active tendon control system. In other areas of applications, investigators have proposed different methods to identify the system time delay that is not a multiple of sampling time and the output measurement is contaminated with noise.[12–16] The approach proposed by Chen *et al.*[15] is particularly interesting, because the results are less sensitive to noise and the sampling interval is adjusted according to the magnitude of the time delay during the identification process. Recently, Baugh[17] proposed the use of temporal-process algebra to develop quantitative models for computational systems to predict the overall timing properties of an active control process (e.g. the time delay between sampling data and signaling an actuator), given the timing properties of each individual component. The models include a description of the control flow for each component, its communication behaviour, and its execution time. In addition to identifying the system time delay, the investigation by Baugh[17] has a tremendous potential to improve the reliability and fault tolerance of active control systems for civil engineering structures.

EFFECT OF FIXED TIME DELAY ON THE STABILITY AND PERFORMANCE

Application of unsynchronized control forces due to time delay may result in a degradation of the control performance and may even render the controlled structure unstable. Hence, the stability analysis of time-delayed control systems is an important aspect of structural control. One of the earliest works related to time-lag in control systems was presented by Callander *et al.*[18] Ansoff[19] studied the stability of linear oscillating systems with constant time-lag; however, his approach was mathematically complicated and difficult to interpret physically. A simpler approach was presented by Satche[20] using the dual Nyquist

criterion and the application of such an approach to time-delayed velocity feedback systems was demonstrated. Weiss[21] presented an extensive bibliography on the subject of transportation lag systems, and Choksy[22] also gave an extensive bibliography on the subject of time-lag systems. A general introduction to the stability of time-lag systems was presented by Choksy,[23] in which both graphical and analytical methods were described. More recently, Malek-Zavarei and Jamshidi[24] discussed different aspects of time-delay problems in detail, e.g. time-domain (Lyapunov) and frequency-domain methods of stability, optimization and control of time-delay systems, etc.

The stability investigation of time-delayed actively controlled systems is relatively recent for civil engineering structures. Roorda[25] showed experimentally the importance of the relationship between the phase-delay in control force and the structural response. He showed that active control adds the largest active damping to the structure, when the feedback control force lags the response by a certain optimum phase angle. Basharkhah and Yao[2] investigated the reliability aspects of structural control by considering the effects of two types of time delays; namely, the fixed time delay (e.g., β_1 in Figure 1) and the time constant (which is related to the build-up time delay β_2 in Figure 1). For a SDOF linear system, they have found that higher values of time delay increase the failure probability of the controlled structures. Their results showed that the time-delayed control force always reduces the reliability of actively controlled civil engineering structures. Abdel-Rohman[26] showed the adverse effect of time delay on the controlled response of structures through an example of a SDOF system equipped with an active tendon system. In an another study, Abdel-Rohman[27] investigated the effect of time delay on control of distributed parameter structures using the Taylor series expansion. However, Marshall[28] has shown that the use of the truncated Taylor series for the stability analysis of a closed-loop system with time delay may yield erroneous results. Yang *et al.*[29] have examined the effect of time delay on control of seismic-excited buildings through extensive numerical simulations results. For an eight-storey shear-beam-type building considered, they demonstrated that the control performance of the closed-loop system always degrades in the presence of time delay and that the effect of time delay on the control performance depends both on the control algorithms and control devices used. Iwan and Hou[30,31] showed the detrimental effect of time delay by demonstrating the existence of infinite number of critical values of time delay for which the structural response becomes unbounded. Carotti and Lio[6] studied the effect of time delay on a SDOF model through the bench test of various components of a control system, and proposed the following empirical equation to maintain the efficacy of active control:

$$\gamma\beta \leqslant 0{\cdot}25T_s \tag{3}$$

where $\gamma \geqslant 3$, $\beta = \beta_1 + \beta_2$ and T_s is the fundamental period of structure. However, equation (3) may be too conservative.

Recently, experimental studies were conducted by Abdel-Mooty and Roorda[5] on the effect of time delay for control of a simply supported beam equipped with a hydraulic actuator. A direct velocity feedback controller was designed by assigning 10 per cent damping to the first mode only. Although significant reduction in the first vibrational mode was observed, instability in higher modes (mainly the second mode) was experienced due to the time delay in the control system. To eliminate instability in higher modes, a low-pass filter with a cutoff frequency around the first mode was introduced. In this case, however, the instability in the second mode became even more severe, because the low-pass filter introduces different gains and phase delays at different frequencies. Consequently, the effect of time delay became more pronounced.

Critical time delay for a SDOF system

Pu and Kelly[32] examined the stability of a SDOF controlled structure with time-delayed control force using the frequency-domain analysis. The equation of motion of a SDOF system is given by

$$\ddot{x}(t) + 2\zeta_s\omega_s\dot{x}(t) + \omega_s^2 x(t) = u(t) + f(t) \tag{4}$$

where ω_s and ζ_s are the natural frequency and damping ratio, respectively, $u(t)$ is the control force, and $f(t)$ is the external excitation. Consider a linear control of the form

$$u(t) = -g_1 x(t) - g_2 \dot{x}(t) \tag{5}$$

where g_1 and g_2 are displacement and velocity feedback gains, respectively. The closed-loop transfer function of the structure can be written as

$$T(s) = \frac{x(s)}{f(s)} = \frac{P(s)}{1 + P(s)F(s)} \tag{6}$$

in which

$$P(s) = (s^2 + 2\zeta_s\omega_s s + \omega_s^2)^{-1}, \qquad F(s) = (g_1 s + g_2) \tag{7}$$

In equations (6) and (7), s is a Laplace transform variable. For a fixed time delay β in the control force $u(t)$, the transfer function $F(s)$ for the feedback part is modified as

$$F_d(s) = e^{-s\beta} F(s) \tag{8}$$

Classically, the relative stability of a system is defined by the gain and phase margins of the open-loop transfer function $P(s)F_d(s)$ in equations (6)–(8).[33,34] Due to the time delay β, a phase delay is introduced in the applied control force, $u(t)$. Consequently, the phase margin of the open-loop transfer function $P(s)F_d(s)$ is reduced by the amount of the phase delay due to the time delay. The time delay at which the phase margin of $P(s)F_d(s)$ becomes zero is defined as the critical time delay, β_{max}. The controlled structure becomes unstable when the time delay β in the control force is larger than β_{max}.

For the transfer function $P(i\omega)F_d(i\omega)$, the phase margin is defined by the angle $\angle P(i\bar{\omega})F_d(i\bar{\omega})$ between the phasor and the negative real axis at the frequency $\bar{\omega}$ at which

$$|P(i\bar{\omega})F_d(i\bar{\omega})| = 1 \tag{9}$$

The frequency $\bar{\omega}$ in equation (9) is called the gain cross-over frequency. Pu and Kelly[32] adopted this concept to propose a graphical method to determine the critical time delay for a SDOF system with the time-delayed control force. Through numerical examples, they showed that the relative stability of a controlled structure depends on the delay time, natural frequency and the amount of active damping introduced by the control force. Because the method is graphical in nature, it is not easy to investigate the effects of structural and control parameters on the critical time delay quantitatively.

Agrawal[35] and Agrawal *et al.*[36,37] presented a stability analysis of a SDOF system with a fixed time-delayed feedback and derived a closed-form solution for the critical time delay. With the control law in equation (5), the equation of motion, equation (4), with time delay β in the control force can be written as

$$\ddot{x}(t) + 2\zeta_s\omega_s\dot{x}(t) + \omega_s^2 x(t) + g_1 x(t-\beta) + g_2\dot{x}(t-\beta) = f(t) \tag{10}$$

Taking Laplace transformation of equation (10), one obtains the characteristics equation

$$H(s) = s^2 + 2\zeta_s\omega_s s + \omega_s^2 + g_1 e^{-s\beta} + g_2 s e^{-s\beta} = 0 \tag{11}$$

The characteristics equation in equation (11) has infinite number of roots due to the exponential term $e^{-s\beta}$. The stability of controlled structure cannot be determined simply by calculating the first few roots of equation (11), since the stability is guaranteed only when all the roots have negative real parts. The system is at the limit of stability when roots of equation (11) are imaginary, i.e. $s = i\omega$. Letting $s = i\omega$ in equation (11) where ω is a real frequency, one obtains

$$-\omega^2 + i2\zeta_s\omega_s\omega + \omega_s^2 + g_1 e^{-i\omega\beta} + i\omega g_2 e^{-i\omega\beta} = 0 \tag{12}$$

The displacement and velocity feedback vectors in equation (12) are shown in Figure 4 in the complex plane. Figure 4 indicates that the feedback forces may contribute to negative active damping, and hence the total

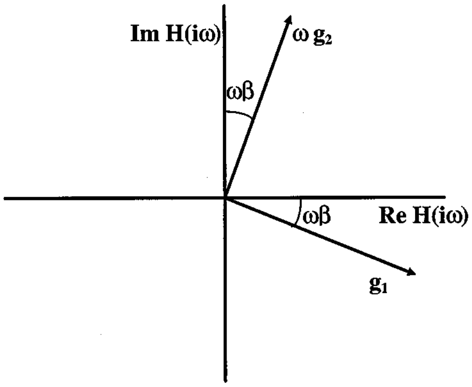


Figure 4. Displacement and velocity feedback vectors in the complex plane

damping of the controlled structure may become negative. When the total damping (active damping plus structural damping) becomes zero, the system is at the limit of stability. Equation (12) can be solved by separating the real and imaginary parts as follows:

$$g_1 \sin(\omega\beta) - g_2\omega\cos(\omega\beta) = 2\zeta_s\omega_s\omega \tag{13}$$

$$g_1 \cos(\omega\beta) + g_2\omega\sin(\omega\beta) = \omega^2 - \omega_s^2 \tag{14}$$

The two roots of ω are obtained from equation (13) and (14) analytically as

$$\omega_1 = \sqrt{\frac{2\omega_s^2 + g_2^2 - 4\zeta_s^2\omega_s^2 + L}{2}}, \qquad \omega_2 = \sqrt{\frac{2\omega_s^2 + g_2^2 - 4\zeta_s^2\omega_s^2 - L}{2}} \tag{15}$$

where

$$L = \sqrt{(g_2^2 - 4\omega_s^2\zeta_s^2)(g_2^2 - 4\omega_s^2\zeta_s^2 + 4\omega_s^2) + 4g_1^2} \tag{16}$$

It can be shown from equtions (15) and (16) that $\omega_2 < \omega_s < \omega_1$. Corresponding to these two values of ω, equation (13) can be solved to obtain two values β_1 and β_2 of the time delay β. It can be shown that the critical time delay β_{max} is[37]

$$\beta_{max} = \beta_1 = \frac{2}{\omega_1}\tan^{-1}\left[\frac{-g_1 + \sqrt{g_1^2 + (g_2^2 - 4\omega_s^2\zeta_s^2)\omega_1^2}}{(g_2 - 2\zeta_s\omega_s)\omega_1}\right] \tag{17}$$

As observed from equation (17), the critical time delay depends on the fundamental frequency ω_s, the damping ratio ζ_s, and the feedback gains g_1 and g_2. Denoting $g_2 = 2\zeta_c\omega_s$ where ζ_c is the added damping ratio, and considering only the velocity feedback ($g_1 = 0$), one can show from equation (17) that the structure cannot be driven unstable for any amount of time delay if $\zeta_c < \zeta_s$. This indicates that the bigger the passive damping ζ_s is, the less serious the time-delay problem will be. For the special case of velocity feedback control of an undamped structure, equation (17) can be reduced to

$$\frac{\beta_{max}}{T_s} = \frac{1}{4(\sqrt{\zeta_c^2 + 1} + \zeta_c)} \tag{18}$$

where T_s is the natural period of uncontrolled structure and ζ_c is the active (added) damping ratio. It is observed from equation (18) that the ratio β_{max}/T_s decreases almost linearly as the active damping is increased, and β_{max} is approximately one-fourth of the natural period T_s for very small active damping. Since

the critical time delay, β_{max}, in equation (18) is directly proportional to the fundamental period of the uncontrolled structure, T_s, β_{max} will be small for structures with short natural periods. Hence, the critical time delay may be very small for MDOF systems if the control force affects higher modes (with short natural periods).

The performance of the controlled structure with time-delayed control force was studied through numerical simulations by Agrawal *et al.*[37] It was found that the control performance degrades significantly only when the time delay is close to the critical time delay, β_{max}, and the performance degradation is insignificant for $\beta/\beta_{max} \leqslant 0{\cdot}6$.

Inaudi and Kelly[38] derived the closed-form solution of the critical time delay for a SDOF system with only velocity feedback (i.e. $g_1 = 0$) using the concept of phase margin as described previously. The phase margin is calculated at the gain cross-over frequency $\bar{\omega}$, and equation (9) can be solved to obtain two positive roots for $\bar{\omega}$. These two roots are the same as ω_1 and ω_2 in equation (15) with $g_1 = 0$. Since the phase margin is zero for the critical time delay, we can write

$$\angle P(\mathrm{i}\omega_1)F_\mathrm{d}(\mathrm{i}\omega_1) = \pm\,\pi \tag{19}$$

where the larger value of $\bar{\omega}$, i.e. $\bar{\omega} = \omega_1$ in equation (15), has been used, because it yields the smaller value of time delay. Using equations (6) and (7), one can solve equation (19) for the critical time delay as

$$\beta_{max} = \frac{1}{\omega_1}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2\omega_1\zeta_s\omega_s}{\omega_s^2 - \omega_1^2}\right)\right] \tag{20}$$

The critical time delay β_{max} in equation (20) is a special case of equation (17), and both yield the same results for the velocity feedback (there are typographical errors for equations (8) and (11) in Inaudi and Kelly[38]).

Hughes and Hong[39] also presented a stability analysis of a SDOF system with time-delayed velocity feedback using the root-locus method. The closed-form solutions for the critical time delay they obtained are similar to equations (19) and (20). Similarly, Lin *et al.*[40,41] also investigated the SDOF system with time-delayed output feedback, and obtained a closed-form solution for the critical time delay for direct velocity feedback. For the case of undamped structures, their result reduces to equation (18), that was derived by Agrawal *et al.*[37]

The critical time delay in equation (17) has been derived by assuming that time delays in displacement and velocity feedbacks are identical. Hou and Ali[42] investigated the stability of SDOF systems with unequal time delays in displacement and velocity feedbacks. Denoting β_x and $\beta_{\dot{x}}$ as time delays in displacement and velocity feedbacks, respectively, equations (13) and (14) can be modified as

$$g_1 \sin(\omega\beta_x) - g_2\omega\cos(\omega\beta_{\dot{x}}) = 2\zeta_s\omega_s\omega \tag{21}$$

$$g_1 \cos(\omega\beta_x) + g_2\omega\sin(\omega\beta_{\dot{x}}) = \omega^2 - \omega_s^2 \tag{22}$$

From equations (21) and (22) it can be shown that there exist infinite pairs of critical time delays β_x and $\beta_{\dot{x}}$ and these pairs can be obtained analytically. Further, Hou and Ali[42] showed that there exist multiple regions of stability in the $\beta_{\dot{x}}$ vs. β_x plane and these regions appear in the form of bands. These multiple stability bands can be exploited to stabilize a system by intentionally introducing appropriate time delays for both β_x and $\beta_{\dot{x}}$ in the feedback loop.

Previous studies described so far have focused on the detrimental effect of time delay. Udwadia and Kumar,[43] Kumar[44] and Kumar and Udwadia[45] investigated the advantages of appropriate choices of time delays for stabilizing non-collocated control of certain classes of classically damped structural systems. They showed that undamped systems, which cannot be stabilized by use of non-collocated sensors with pure velocity feedback, can be stabilized by using time-delayed velocity feedback control. Further, they obtained easy-to-compute analytical expressions for the bounds on time delays in order to maintain the small gain stability. These results are important for velocity feedback control of large structures.

STABILITY OF TIME-DELAYED MDOF SYSTEM

The equation of motion for an n-degree-of-freedom building structure with a fixed time delay β in the control force and subject to a horizontal earthquake ground acceleration $\ddot{x}_0$ can be written as

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = \xi U(t - \beta) + D\ddot{x}_0(t) \tag{23}$$

in which $X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is an n-vector with $x_i(t)$ being the interstorey drift of the ith storey unit, M, C and K are $(n \times n)$ mass, damping and stiffness matrices, respectively, $U(t) = [u_1, u_2, \ldots, u_r]^{\mathrm{T}}$ is an r-control vector, ξ is the location matrix of the controller, and D is the earthquake excitation influence vector. In the state-space, equation (23) can be written as

$$\dot{Z}(t) = AZ(t) + BU(t - \beta) + E(t) \tag{24}$$

where Z is a $2n$ state vector, A is a $(2n \times 2n)$ system matrix, B is a $(2n \times r)$ location matrix, and $E(t)$ is $2n$ vector given by

$$Z(t) = \begin{bmatrix} X \\ \dot{X} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}\xi \end{bmatrix}, \quad E(t) = \begin{bmatrix} 0 \\ M^{-1}D\ddot{x}_0(t) \end{bmatrix} \tag{25}$$

In general, a linear feedback controller, either a full-state or static output feedback, is designed by considering the system in equation (24) without a time delay, i.e.

$$U(t) = -GZ(t) = -G_1X(t) - G_2\dot{X}(t) \tag{26}$$

where G is a $(r \times 2n)$ constant feedback gain matrix, and G_1 and G_2 are $(r \times n)$ submatrices.

Approximate approach

Unlike SDOF systems, it is difficult to derive a closed-form solution for $\beta_{\max}$ for MDOF systems. Pu and Kelly[32] and Agrawal *et al.*[37] have suggested an approximate method by decoupling the equation of motion on the left-hand side of equation (23) into the modal co-ordinates and neglecting the coupling effect among different modes due to feedback control. Then, the critical time delay for the ith mode, $\beta_{i\max}$, can be calculated using the closed-form solution for SDOF system in equation (17). The minimum of $\beta_{i\max}$ (for $i = 1, 2, \ldots, n$) can be considered as the critical time delay, $\beta_{\max}$, for the entire MDOF system. Since the applicability and limitations of this approximate method have not been investigated, numerical simulation results will be presented later for demonstration purposes.

Assume that the uncontrolled structure in equation (23) is classically damped so that the normal modes exist. Let ω_i and ζ_i be the ith natural frequency and damping ratio of the uncontrolled structure in the ith mode, respectively, and Γ be the $(n \times n)$ real modal matrix consisting of n modeshapes. Further, let $X = \Gamma Y$ where $Y = [y_1, y_2, \ldots, y_n]^{\mathrm{T}}$ is the modal vector. Substituting $X = \Gamma Y$ and equation (26) into equation (23) and using the orthogonal condition, one obtains the equation of motion for each mode, say, the ith mode, as follows:

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i + g_{1i}y_i(t - \beta) + g_{2i}\dot{y}_i(t - \beta) = f_{ni}(t), \quad i = 1, 2, \ldots, n \tag{27}$$

In equation (27), g_{1i} and g_{2i} are the ith diagonal elements of the full matrices $\Gamma^{\mathrm{T}}M^{-1}\xi G_1\Gamma$ and $\Gamma^{\mathrm{T}}M^{-1}\xi G_2\Gamma$, respectively, and f_{ni} is the generalized force in the ith mode. In deriving equation (27), all the coupling terms among different modes due to the feedback control, equation (26), have been neglected, i.e. all the off-diagonal terms of matrices $\Gamma^{\mathrm{T}}M^{-1}\xi G_1\Gamma$ and $\Gamma^{\mathrm{T}}M^{-1}\xi G_2\Gamma$ have been neglected. Since equation (27) is similar to equation (10), the critical time delay $\beta_{i\max}$ can be calculated by using equations (15) to (17).

As will be demonstrated later, this approximate approach may give reasonable values of $\beta_{\max}$ when the modes of the structure are widely spaced. However, when vibrational modes of the structure are very close to each other, e.g. control of a building using an active tuned-mass damper, this approximate approach may yield erroneous and unreliable values for the critical time delay. Hence, in the next subsection, we present

theoretical approaches for the calculation of the critical time delay for MDOF systems equipped with a single controller or multiple controllers. Applications of these approaches will be demonstrated through simulation results.

Exact solutions

The stability of the system in equation (24) using a linear controller in equation (26) has been studied in other areas of engineering disciplines. Substituting the controller in equation (26) into equation (24) and neglecting the external excitation $E(t)$, the closed-loop system becomes

$$\dot{Z}(t) = AZ(t) + \bar{B}Z(t-\beta) \tag{28}$$

where $\bar{B} = -BG$. Malek-Zavarei and Jamshidi[24] have discussed different aspects of stability of equation (28) based on the Lyapunov theorem of stability. Su *et al.*[46] proposed an approach to determine β_{max} for the constant time delay, β. Similarly, when β is assumed to be time varying, Su[47] proposed a method for determining β_{max} for a system with uncertainties in the matrices A and $\bar{B}$. The methods mentioned above are based on the Lyapunov approach of stability and the results are too conservative to be of practical use for civil infrastructure. This will be demonstrated in the next section by simulation results.

To determine the critical time delay β_{max} accurately, we present two approaches in the following. The first approach is based on the concept of gain and phase margins of the open-loop transfer function, which is applicable only to structures equipped with a single actuator (controller). The second approach is based on the evaluation of the limiting solutions of the characteristics equation. This approach is applicable to structures equipped with either a single actuator or multiple actuators.

Single actuator case. Taking Laplace transformation of equations (24) and (26), one obtains the open-loop transfer function, $T(s)$, from the control force $U(t)$ to the state vector $Z(t)$ as follows:

$$T(s) = P(s)F_d(s) = G(sI-A)^{-1}B\mathrm{e}^{-s\beta} \tag{29}$$

For a MDOF system equipped with a single actuator, the transfer function in equation (29) is a scalar. Hence, the gain cross-over frequency $\bar{\omega}$ can be obtained from

$$|T(\mathrm{i}\bar{\omega})| = |G(\mathrm{i}\bar{\omega}I-A)^{-1}B\mathrm{e}^{-\mathrm{i}\bar{\omega}\beta}| = 1 \tag{30}$$

Since $|\mathrm{e}^{-\mathrm{i}\bar{\omega}\beta}| = 1$, equation (30) can be written as

$$|G(\mathrm{i}\bar{\omega}I-A)^{-1}B| = 1 \tag{31}$$

For a SDOF system, equation (31) can be solved analytically for $\bar{\omega}$ as described previously, equations (19) and (20). For MDOF systems, equation (31) represents a general polynomial equation and $\bar{\omega}$ can only be solved numerically. Then, following the definition of the phase margin, the time delay $\bar{\beta}$ that results in zero phase margin can be calculated from

$$\angle G(\mathrm{i}\bar{\omega}I-A)^{-1}B - \bar{\omega}\bar{\beta} = \pm\pi \tag{32}$$

Thus, the solution of the time delay $\bar{\beta}$ at zero phase margin is obtained from equation (32) as follows:

$$\bar{\beta} = \frac{1}{\bar{\omega}}[\angle G(\mathrm{i}\bar{\omega}I-A)^{-1}B \pm \pi] \tag{33}$$

In general, equations (31) and (33) have infinite pairs of solutions for $\bar{\omega}$ and $\bar{\beta}$ corresponding to the time when poles of the closed-loop system are either entering or leaving the right-half complex plane. Since MDOF systems have many pairs of complex-conjugate poles, each pair of $\bar{\omega}$ and $\bar{\beta}$ corresponds to a situation when a pair of complex-conjugate poles is either entering or leaving the right-half complex plane. When the open-loop system, A, and the closed-loop system without time delay, (A-BG), are stable, the smallest value of

$\bar{\beta}$ obtained from equations (31) and (33) is the critical time delay β_{max}. The above approach for determining the critical time delay can be applied to both full-state and static output feedback control. Inaudi and Kelly[38] presented a similar approach only for the special case of velocity feedback.

Multiple actuator case. For a single actuator or multiple actuators case, the critical time delay β_{max} can be determined from the characteristic equation $H(s)$ of the closed-loop system, i.e.,

$$H(s, \beta) = |sI - A + BG\mathrm{e}^{-s\beta}| = 0 \tag{34}$$

The closed-loop system is at the limit of stability when a pole becomes imaginary, i.e., $s = \mathrm{i}\omega$. Replacing s by $\mathrm{i}\omega$, equation (34) becomes

$$H(\mathrm{i}\omega, \beta) = |\mathrm{i}\omega I - A + BG\mathrm{e}^{-\mathrm{i}\omega\beta}| = 0 \tag{35}$$

Equating the real and imaginary parts of equation (35) to zero, one obtains two simultaneous non-linear equations,

$$\mathrm{Re}\,[H(\mathrm{i}\omega, \beta)] = 0, \qquad \mathrm{Im}\,[H(\mathrm{i}\omega, \beta)] = 0 \tag{36}$$

Two unknowns, ω and β, can be obtained by solving these two simultaneous non-linear equations by iterations. The solution for the smallest value of β is the critical time delay, β_{max}, and the corresponding frequency ω is denoted by ω^*. It has been found through numerical simulations that the solution of equation (36) is very sensitive to the initial trials of ω and β. In general, for a stable open-loop system, $\omega = 0$ and $\beta = 0$ can be a reasonable initial trial.

It is mentioned that the formulation above for the stability analysis of controlled systems with time delay applies to continuous-time systems only. For control systems equipped with digital computers with sampling times, the stability should be performed based on the discrete difference equation of motion. However, when the sampling rate (frequency) of the control system is sufficiently high, the stability analysis method presented above can be used to calculate the critical time delay.

NUMERICAL EXAMPLES FOR MDOF SYSTEMS

Example 1: Two DOF system with single controller

A SDOF structure equipped with an active tuned-mass damper (ATMD) is considered. Let m_s, ζ_s and ω_s be the mass, damping ratio and frequency of the structure, respectively. Further, let $m_d = \mu m_s$, $\omega_d = \eta\omega_s$ and $\zeta_d = \alpha\zeta_s$ be the respective quantities of the mass damper, where μ, η and α are the mass ratio, frequency ratio, and damping ratio of the mass damper with respect to the structure. The effect of time delay is investigated for a structure with $m_s = 1000$ tons, natural period of 3.0 sec ($\omega = 2{\cdot}094$ rad/sec), structural damping ratio of $\zeta_s = 1$ per cent, $\mu = 5$ per cent, $\eta = 1{\cdot}0$, and $\alpha = 12{\cdot}72$. A linear controller, $U(t) = -GZ(t)$, has been designed by the LQR method such that the damping ratios in two modes of the closed-loop system without time delay are $\zeta_{c1} = \zeta_{c2} = 20$ per cent. Based on the exact method of analysis presented above, the critical time delay, β_{max}, for the closed-loop system is calculated from equation (36) as $\beta_{max} = 0{\cdot}361$ sec. The critical time delay based on the Lyapunov method proposed by Su *et al.*[46] and Su[47] has also been calculated as $\beta_{max} = 0{\cdot}056$ sec for constant β, and $\beta_{max} = 0{\cdot}055$ sec when β is assumed to be unknown time varying and unbounded. These values of the critical time delay are too conservative to be of practical use as compared with $\beta_{max} = 0{\cdot}361$ sec obtained by the method presented in the previous section.

The effect of time delay on the response of the structure is studied by calculating the H_∞ norm of the relative displacement of the structure, denoted as $H_\infty(\beta)$,

$$H_\infty(\beta) = \max_{\omega}\,[(-\,\omega^2 + \mathrm{i}\omega\bar{C} + \bar{K} + \bar{H}G\mathrm{e}^{-\mathrm{i}\omega\beta})^{-1}M^{-1}J] \tag{37}$$

where J is defined as $J = [1, 0]^T$, indicating a sinusoidal excitation with the frequency ω, $\bar{C} = M^{-1}C$, $\bar{K} = M^{-1}K$ and $\bar{H} = M^{-1}\xi$. Figure 5 shows the plot of the ratio $H_\infty(\beta)/H_\infty(0)$ as a function of the time-delay ratio β/β_{max} with $\beta_{max} = 0{\cdot}361$ sec. It is observed from Figure 5 that there is no significant degradation in the performance of the control system (solid curve) up to a certain time delay. However, as the time delay approaches the critical time delay, $\beta_{max} = 0{\cdot}361$ sec, i.e. $\beta/\beta_{max} \to 1$, the degradation of the control performance increases drastically, and the system becomes unstable when $\beta/\beta_{max} = 1$.

Under the El Centro NS (1940) earthquake excitation, the effect of time delay on the performance the ATMD system is presented in Figure 6. Figures 6(a) and 6(b) show the plots of the peak-response ratio $x(\beta)/x(0)$ and the peak-control force ratio $u(\beta)/u(0)$ vs. the time-delay ratio β/β_{max}, where $x(\beta)$ denotes the peak relative displacement of the structure, $u(\beta)$ indicates the peak-control force, and $\beta_{max} = 0{\cdot}361$ sec. It is observed from Figure 6(b) that the peak-control force increases with respect to the increase of time delay β. However, a significant degradation for the peak response occurs only when the time delay β exceeds 60 per cent of the critical time delay β_{max}, i.e. $\beta > 0{\cdot}6\,\beta_{max}$, as indicated by Figure 6(a).

The natural frequencies of the 2DOF system depend on the tuning ratio η of the mass damper. To investigate the influence of the tuning ratio (or natural frequencies) on the critical time delay β_{max}, different cases of the tuning ratio η are considered in the following. Various linear controllers, $U(t) = -GZ(t)$, have been designed by the LQR method for different values of η. Weighting matrices Q and R for different linear controllers are chosen such that (i) the damping ratios for both modes of the closed-loop system without time delay are identical, i.e. $\zeta_{c1} = \zeta_{c2}$, and (ii) $\zeta_{c1} = \zeta_{c2} = 10$, 20 and 30 per cent. The critical time delays β_{max} based on (i) the exact analysis method presented in equation (36) and (ii) the approximate analysis method by decoupling the equations of motion into the modal co-ordinates and neglecting the coupling effect presented in equation (27), are shown in Table I for linear controllers designed above. It is observed that the critical time delays, β_{max}, obtained by using the approximate and exact analysis methods, match reasonably well for all the three levels of the closed-loop damping for $\eta = 0{\cdot}4$ and $1{\cdot}6$. However, for the tunning ratios $\eta = 0{\cdot}8$ or $1{\cdot}0$, the difference between approximate and exact solutions for β_{max} becomes significant. This happens because the coupling between the structural mode and the TMD mode increases as η approaches $1{\cdot}0$, i.e. two modes are close to each other. Consequently, the critical time delay β_{max} calculated based on the approximate analysis method is not reliable for structures with closely spaced vibrational modes.

Since the effect of time delay is to reduce the phase margin of an actively controlled structure, plots of the phase margin ($\omega^*\beta_{max}$) based on the exact solution vs. the tuning ratio (η) are presented in Figure 7 for three levels of the closed-loop damping above, i.e. $\zeta_{1c} = \zeta_{2c} = 10$, 20 and 30 per cent. It is observed that as η approaches 1, i.e. the frequency of TMD is tuned to that of the structure, the phase-margin ($\omega^*\beta_{max}$)

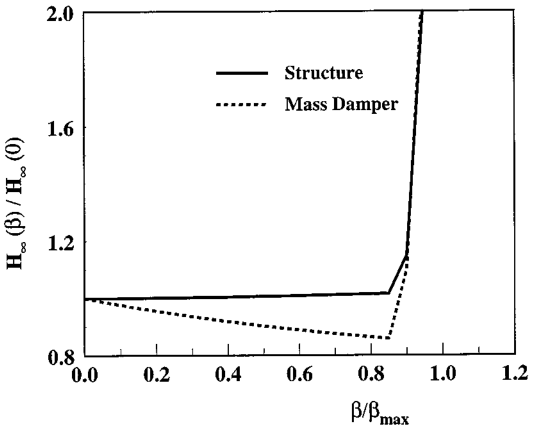


Figure 5. Response ratio $H_\infty(\beta)/H_\infty(0)$ vs. time-delay ratio β/β_{max} for a 2DOF system

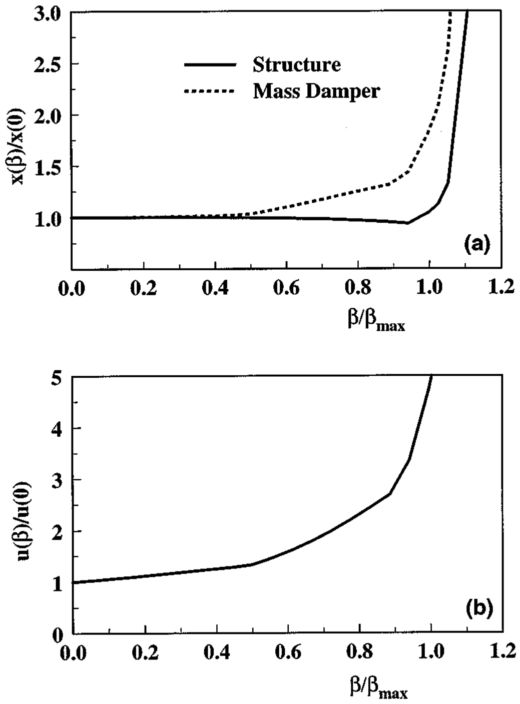


Figure 6. Peak response ratio $x(\beta)/x(0)$ and peak control force ratio $u(\beta)/u(0)$ vs. time-delay ratio β/β_{max} for a 2 DOF system; (a) $x(\beta)/x(0)$, and (b) $u(\beta)/u(0)$

Table I. Critical time-delay, β_{max}, by approximate and exact analysis methods for 2DOF System

Tuning ratio (η)	$\zeta_{c1} = \zeta_{c2}$ (%)	Critical time-delay, β_{max} (s): Approximate solution, equation (27)	Critical time-delay, β_{max} (s): Exact solution, equation (36)
(1)	(2)	(3)	(4)
0·4	10	0·633	0·632
0·4	20	0·480	0·492
0·4	30	0·375	0·396
0·8	10	0·461	0·523
0·8	20	0·278	0·400
0·8	30	0·189	0·320
1·0	10	0·374	0·468
1·0	20	0·209	0·361
1·0	30	0·139	0·289
1·6	10	0·370	0·374
1·6	20	0·250	0·278
1·6	30	0·181	0·221

decreases and it is minimum for $\eta = 1$. Moreover, the phase margin decreases as the damping of the closed-loop system is increased from 10 to 30 per cent. Hence, the time-delay problem is more serious for systems with either closely spaced vibrational modes or higher closed-loop dampings.

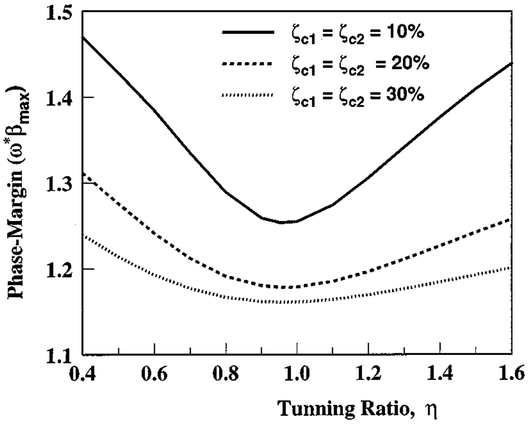


Figure 7. Phase-margin, $\omega^*\beta_{max}$, vs. the tuning ratio, η

Example 2: Three-storey building with a single controller

Consider the three-storey experimental building model equipped with an active tendon controller in the first storey unit used by Chung *et al.*[48] The natural frequencies of this building are 2·24, 6·83 and 11·53 Hz, and the corresponding modal damping ratios are 1·62, 0·39 and 0·36 per cent, respectively. The tendon stiffness is $k_c = 371{\cdot}95$ kN/m (2124 lb/in) and inclination of tendons is $\theta = 36°$. The LQR control law is used to design the controller by choosing the state and control weighting matrices as follows:

$$Q = \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix}, \qquad R = 20k_c \tag{38}$$

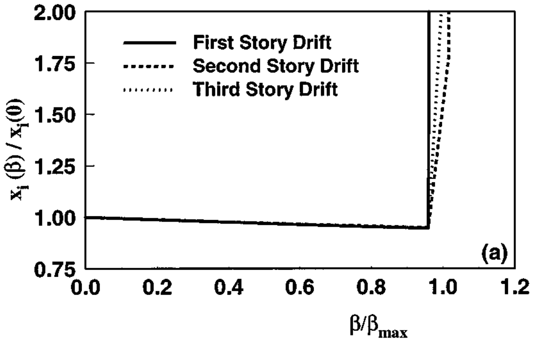
in which K is the stiffness matrix of the building given in Chung *et al.*[48] The peak response quantities for the building subject to the El Centro NS (1940) earthquake excitation are presented in Figure 8. Figure 8(a) shows the plots of the peak interstory drift ratios $x_i(\beta)/x_i(0)$ vs. β/β_{max} for $i = 1, 2, 3$, whereas Figure 8(b) shows the peak-control force ratio $u(\beta)/u(0)$ vs. β/β_{max}. In the notations above, $x_i(\beta)$ and $u(\beta)$ denote the ith peak interstorey drift and the peak-control force, respectively, for a time delay β, and $\beta_{max} = 17{\cdot}772$ msec is obtained from the exact solution of equation (36). The critical time delay β_{max}, based on the approximate solution in equation (27), is 17·537 msec. It is observed that the performance of the controlled structure remains almost unaffected by the time delay up to 0·98 β_{max} and the structure becomes unstable when $\beta = \beta_{max}$.

Example 3: Three-storey building with two actuators

Consider the same 3DOF experimental building model discussed in Example 2. Two active tendon controllers are installed; one in the first-storey unit and another in the second-storey unit. The stiffness of tendons, k_c, and the inclination of tendons θ, are identical to that in Example 2. The LQR control law is used to design the controllers by choosing the state and control weighting matrices as

$$Q = \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix}, \quad R = k_c I_r \tag{39}$$

in which K is the stiffness matrix of the building and I_r is a (2×2) identity matrix. Since the structure is equipped with two actuators, the exact solution for the critical time delay is calculated from equation (36); with the result $\omega^* = 90$ rad/sec and $\beta_{max} = 14{\cdot}674$ msec. The critical time delay has also been calculated



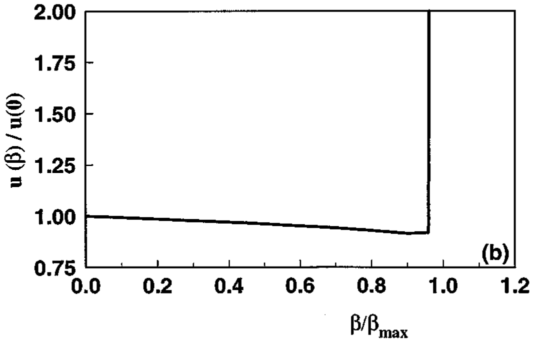


Figure 8. Peak interstorey drifts and peak control force for the three-storey building; (a) $x_i(\beta)/x_i(0)$ vs. β/β_{max}, and (b) $u(\beta)/u(0)$ vs. β/β_{max}

based on the approximate method of analysis in equation (27); with the result $\omega^* = 86{\cdot}19$ rad/sec and $\beta_{max} = 14{\cdot}514$ msec. This shows that the critical time delay β_{max} obtained using the approximate analysis method is quite reasonable. However, the exact solution should be used in the situations where either the structure has non-proportional damping or closely spaced modes, for instance, with the installation of an ATMD.

CONCLUSIONS

A state-of-the-art review for the effect of the fixed time delay on actively controlled civil engineering structures subject to seismic excitation has been presented. This includes the identification of time delay, the effect of time delay on the stability and performance of the controlled structures, and the evaluation of the critical time delay. In particular, a critical review of the approximate analysis method currently available for the determination of the critical time delay for Multiple-Degree-of-Freedom (MDOF) systems has been conducted and new simulation results have been presented to demonstrate its limitations in practical applications. Further, a method of stability analysis for the critical time delay of MDOF systems equipped with single or multiple actuators has been presented along with the simulation results to illustrate its applications to seismic hazard mitigations.

Under earthquake excitations, simulation results for the structural response indicate that the degradation of the control performance due to the fixed time delay is not significant until the time delay is close to the critical time delay. It is demonstrated that the time-delay problem is more serious for structures with closely spaced vibrational modes, such as a building equipped with an ATMD. Further, for structures with closely spaced vibrational modes, the critical time delay obtained based on the approximate method of analysis

currently available in the literature deviate significantly from the exact solution. Consequently, in such a circumstance, the exact-analysis method presented in this paper should be used. A critical review of the methods of compensating the system time delay in the design of controllers along with a new compensation method will be presented later.

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